

**Semester Two Examination, 2021**

**Question/Answer Booklet**

**MATHEMATICS**

**METHODS**

**ATAR Year 12**

**Section One:**

**Calculator-free**

Student Name: **SOLUTIONS**

Please circle your teacher’s name

**Teacher: Miss Hosking Miss Rowden**

**Time allowed for this paper**

Reading time before commencing work: 5 minutes

Working time for paper: 50 minutes

**Materials required/recommended for this paper**

***To be provided by the supervisor***

Number of additional

answer booklets used

(if applicable):

This Question/Answer Booklet

Formula Sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Suggested working time (minutes) | Marks available | Percentage of examination |
| Section One:  Calculator free | 8 | 8 | 50 | 51 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 97 | 65 |
|  |  |  |  | **Total** | 100 |

**Instructions to candidates**

1. The rules for the conduct of the ATAR course examinations are detailed in the *Year 12 Information Handbook 2021*. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Supplementary pages for the use planning/continuing your answer to a question have been provided at the end of the Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free 35% (51 Marks)**

This section has eight (8) questions. Answer **all** questions. Write your answers in the spaces

provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 (5 marks)

a) Determine . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü antiderivative  ü includes constant of integration |

(b) The line intersects the curve at and . Determine the area trapped between line and the curve. (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ writes correct integral  ü antidifferentiates correctly  ü substitutes and simplifies |

Question 2 (5 marks)

A summary of the lengths of a large sample of nails from a production line are shown below.

|  |  |
| --- | --- |
| Length, mm | Relative frequency |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

(a) What proportion of nails are longer than mm? (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct proportion |

(b) Determine the probability that a randomly selected nail from the production line is longer than mm given that it is no longer than mm. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates use of correct relative frequencies  ü simplifies to proper fraction |

(c) State, with reasons, whether the data suggests that the nail lengths are normally distributed. (2 marks)

|  |
| --- |
| Solution |
| Not normally distributed. The relative frequencies do not reflect the bell shaped outline of a normal distribution and appear closer to a uniform distribution. |
| Specific behaviours |
| ✓ states no  ü justifies response |

Question 3 (7 marks)

The curve has one stationary point.

(a) Obtain expressions for and . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ first derivative  ü second derivative |

(b) Determine the coordinates of the stationary point and determine its nature. (4 marks)

|  |
| --- |
| Solution |
| Stationary point at .  Hence stationary point is a maximum. |
| Specific behaviours |
| ✓ equates first derivative to zero and solves  ü calculates coordinates  ü evaluates second derivative at point  ü states maximum |

(c) Explain why the curve has no point of inflection. (1 mark)

|  |
| --- |
| Solution |
| There is no value of for which . |
| Specific behaviours |
| ✓ explains using second derivative |

Question 4 (7 marks)

(a) Let .  
Express as a function of and hence evaluate .

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct antiderivative  ü correct function  ü evaluates |

(b) Let .

(i) Show that . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ shows correct and  ü shows correct structure of quotient rule |

(ii) Hence, or otherwise, evaluate . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct antiderivative  ü evaluates |

Question 5 (7 marks)

(a) By first using log laws, or otherwise, determine in simplest form.

(3 marks)

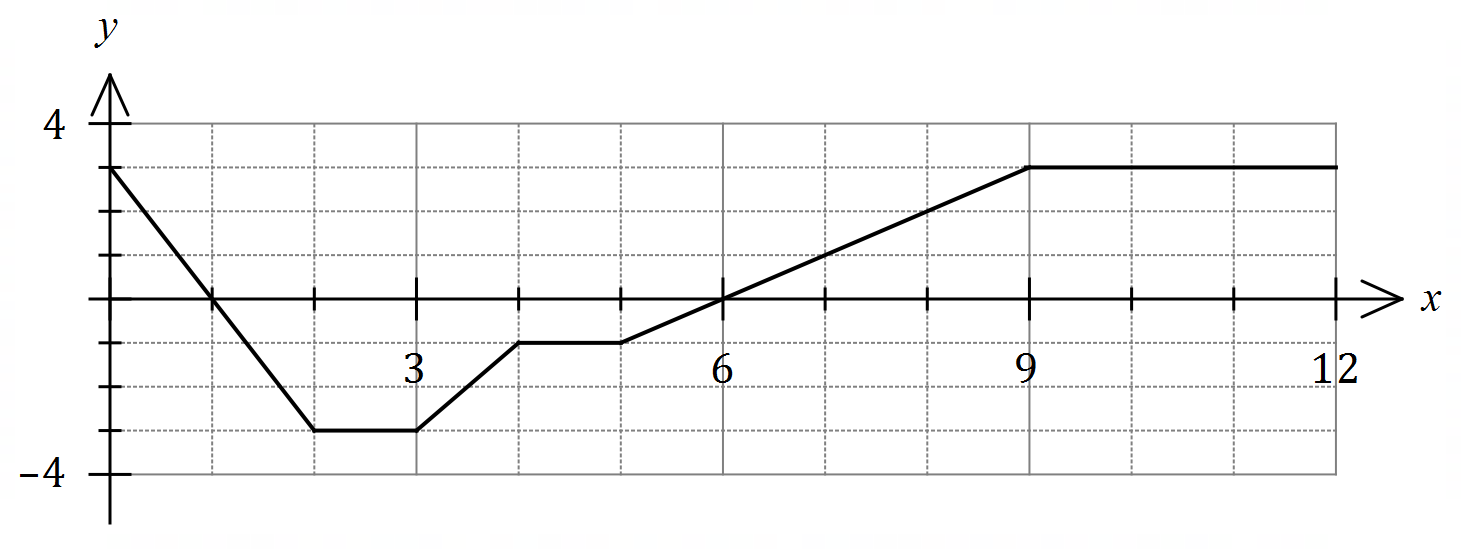
|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ uses one log law appropriately  ü uses second log law appropriately  ü correctly differentiates (and simplifies to either of two forms shown) |

(b) The function for has one stationary point, a global minimum.  
  
Determine the minimum value of the function. (4 marks)

|  |
| --- |
| Solution |
| Stationary when:  Minimum value: |
| Specific behaviours |
| ✓ uses product rule correctly  ü obtains derivative  ü obtains root of derivative  ü calculates minimum value |

Question 6 (6 marks)

The graph of consists of line segments, as shown below.



Evaluate each of the following:

(a) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(b) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates use of signed area  ü correct value |

(c) . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates use of additivity  ✓ determines integral of 2 between 0 and 9  ü correct value |

Question 7 (8 marks)

The random variable is defined by

(a) Determine the value of the constant . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ equation for  ü correct value |

(b) Calculate the expected value of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates  ü correct |

The Bernoulli random variable is solely dependent on , so that when , and for all other values of .

(c) Determine

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct probability |

(i) . (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value |

(ii) . (1 mark)

(iii) . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓  ü |

Question 8 (6 marks)

In triangle , the length of the side opposite angle is given by cm.

Use the increments formula to calculate the approximate change in length of as the size of angle decreases from to .

|  |
| --- |
| Solution |
| When :    Hence length decreases by approximately cm. |
| Specific behaviours |
| ✓ indicates use of chain rule  ü correct derivative  ü evaluates derivative at initial angle  ü indicates incremental change  ü uses increments formula  ü states decrease in length with units |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_